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REGARDING THE ARTICLE BY B. A. LUGOVTSOV "DETERMINATION OF THE MAIN FLOW PARAMETERS IN A SWIRL SPRAYER BY MEANS OF CONSERVATION LAWS" [1]

G. Yu. Stepanov

B. A. Lugovtsov examined the flow of an ideal incompressible fluid through a swirl sprayer with a cylindrical outlet. Figure 1 presents a sketch of the nozzle, with slight modifications from the unit depicted in Fig. 3 of [1]. Walls 1–1 and 2–2 are infinitely distant. The twisting of the flow is of a potential nature ($v_\varphi = \Gamma/(2\pi r)$, $\Gamma = \mathrm{const}$), the twist parameter $A = R\Gamma/(2Q) = \mathrm{const}$, and the free surface of the hollow core of the vortex is monotonic (without standing waves). The pressure $p_2 = \mathrm{const} = 0$ in the meridional section.

We use the Bernoulli integral (incorrectly referred to in [1] as the energy integral) on the free surface at $z = -\infty$ and $z = \infty$

$$v_{1\varphi}^2/2 = v_{2\varphi}^2/2 + v_{2z}^2/2 = B = \mathrm{const}$$

to obtain the discharge coefficient $\mu = Q/(\pi R^2\sqrt{2B})$ and $\overline{R}_1 \equiv R_1/R = A\mu$ as functions of A and $\overline{R}_2 = R_2/R$. The second of these functions is shown by the solid curves in Fig. 2. However, single-valued dependences of μ, $\overline{R}_1$, and $\overline{R}_2$ on A are seen in experiments. In Figs. 2 and 4, ∇ represents maxima on the curves.

G. N. Abramovich in 1943 and (independently) J. Taylor in 1948 proposed that a flow with a maximum discharge coefficient $\mu(\overline{R}_2)$ is realized for each specified parameter A [principle of maximum discharge (PMD)].* Here,

$$2\overline{R}_2^4 - A^2(1-\overline{R}_2^2)^3 = 0, \quad \mu = (1-\overline{R}_2^2)^{3/2}(1+\overline{R}_2^2)^{-1/2}.$$

As is known from the hydraulic theory of spillways with a wide ramp and the linear problem of the fracture of a dam on a horizontal base, the PMD corresponds to the critical flow and follows from the continuity and Euler equations. It can be shown by analogy that if we assume that the thickness h of the layer of liquid in the nozzle outlet is small and the surface of the core of the vortex approximates the cylindrical surface of the outlet in the outlet section, the flow should be critical and have the Froude number

$$\mathrm{Fr} \equiv v_{2z}\Big/\sqrt{hv_{2\varphi}^2/R} = 1 + O(\overline{h}^{3/2}), \quad \overline{h} \equiv h/R \equiv 1 - \overline{R}_2 = (2A)^{-2/3} + O(A^{-4/3}),$$

which for the specified value of A corresponds (to within quantities of the order of $\overline{h}^{5/2}$) to the maximum discharge coefficient μ.

For fairly large, realistic twist parameters ($A \geq 2$), use of the PMD in the hydraulic approximation has solid theoretical support and is backed by numerous experimental studies and is clearly the main technique employed in the design of centrifugal nozzles, various cyclone units, and other pieces of equipment whose operation involves swirling of the flow (for an example, see [2, Sec. 33; 3, pp. 90-94]).

*In Declaration No. 389 on 10.18.90, the State Commission on Inventions recognized G. N. Abramovich, L. A. Klyachko, I. I. Novikova, and V. I. Skobelkina as having discovered the "Law of fluid discharge in a swirled flow," in January of 1948.

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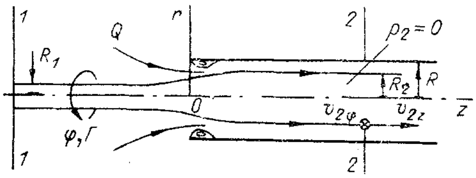

Fig. 1

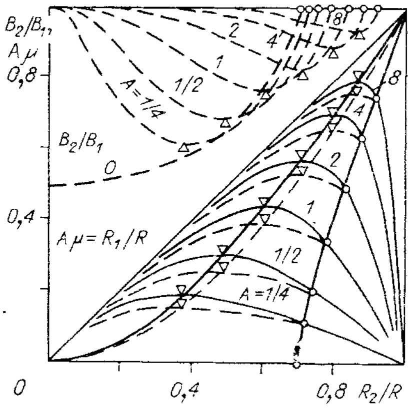

Fig. 2

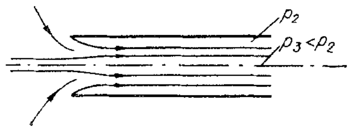

Fig. 3

However, Lugovtsov considers the PMD to be unsubstantiated and even incorrect and asserts that the main parameters of the flow through a centrifugal nozzle of the given design can be determined unambiguously if, along with the Bernoulli integral, the investigator uses the "law of conservation of momentum" (theorem on the change of momentum) in a projection on the z axis (Eqs. (15) in [1]). The result is actually a single-valued dependence of $\bar{R}_1 = A\mu$ on $\bar{R}_2$. This dependence is represented by circles in Fig. 2 for different values of A. The relation just referred to differs markedly from the relation constructed by Abramovich (the solid line passing through the maxima of $\mu(R_2)$).

In constructing Eq. (15), Lugovtsov assumed that "the flow is a potential flow, except for the region with closed streamlines which forms on the interior of the nozzle edge, where the flow structure does not affect subsequent discussion" [1, p. 233]. This view proved to be erroneous. A suction force exists at sharp edges in a continuous potential flow, and this force should be incorporated into Eq. (15). It is absent in the alternative jet potential flow. The latter flow separates from the edges of the nozzle, forming a jet with free boundaries (Fig. 3) under the pressure $p = p_2 = 0$. In the presence of twisting ($A \neq 0$), a hollow core characterized by negative pressure $p_3 = p_1 < p_2$ should extend along the entire axis of this jet. Such a flow clearly cannot be realized in a centrifugal nozzle, and only in the absence of swirling ($A = 0$) do we obtain the familiar flow in a Borda mouthpiece ($\mu = 1/2$, $\bar{R}_2 = \sqrt{2}/2$) – the only point for which the solution in [1] has a physical meaning!

The flow in the nozzle depicted in Fig. 1 can always be calculated if we omit the requirement that the entire flow be a potential flow, ignore friction on the walls of the nozzle, and adopt an additional hypothesis on the distribution of velocity at the outlet of the nozzle ($z = \infty$). Similar solutions, used mainly in connection with the theorem of momentum change, have long been known. Here, it will be sufficient to recall the problem of a sudden expansion of the flow in a cylindrical channel or the problem of hydraulic jumps. More complicated examples can be found in [2, Secs. 32, 52, 53; 3, pp. 36, 50, 76, 97-102].

We will assume that after separation at the inlet edges of the nozzle, the flow reattaches to its wall and, due to a viscosity that is as low as is desired, moves uniformly through the channel ($v_z(r) = \text{const}$). However, the potential twist $\Gamma = 2\pi r v_\varphi = \text{const}$ is conserved along the entire flow. This entails a reduction the Bernoulli constant, $B_2 < B_1 \equiv B$. Then we can easily use the momentum change theorem to find the dependence of $\bar{R}_1 = A\mu$ and B_2/B_1 on A and $\bar{R}_2$. These dependences are shown by the solid lines in Fig. 2. The minima $B_2/B_1(\bar{R}_2)$ coincide with the maxima $\bar{R}_1(\bar{R}_2)$ [or $\mu(\bar{r}_2)$], which can be considered additional

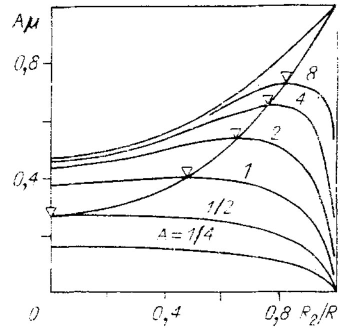

Fig. 4

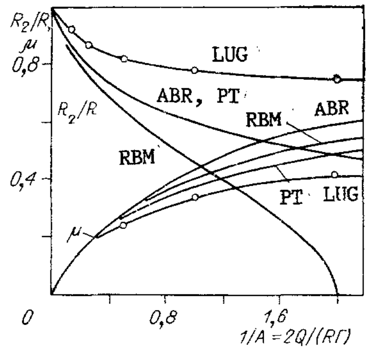

Fig. 5

proof of the PMD; the subseqeunt increase in $\overline{R}_2$ corresponds to the origination of a reverse hydraulic pump in the nozzle (a pseudo shock wave), which is precluded by the second law of thermodynamics. The curves are drawn to the points $B_2/B_1 = 1$, which correspond to the soltuion in [1].

The hypothesis of conservation of the potential swirl is valid for a low-viscosity fluid, since $v_z'(r) \gg v_\varphi'(r)$ after separation at the edges. However, strictly speaking, the effect of viscosity should be to make the incoming flow more like rigid-body motion (RBM) with $v_\varphi = \omega r$. If we ignore the friction on the walls of the nozzle, we can find angular velocity ω from the integral law of conservation of momentum relative to the z axis. (In principle, the effect of friction on the walls of the nozzle may be reduced significantly by allowing its free rotation about the z axis.) Again using the theorem on the change in momentum along the z axis, we find the relation $A\mu(\neq\overline{R}_1)$ corresponding to RMB at the nozzle outlet (Fig. 4). It is worth noting that, in contrast to the previous case (see Fig. 2), for all $A = \text{const} > 0$ and $\overline{R}_2 = 0$, the discharge coefficient $\mu > 0$. In other words, the effect of viscosity is to increase μ for the given A and $\overline{R}_2$.

Minimum discharge regimes ($\mu(\overline{R}_2) = \max$) exist only when the flow is subjected to a sufficiently large twist (in the present case, when $A \geq 1/2$). This fact was established theoretically in 1985 for turbulent flows by Gol'dshtik (see [3, p. 178]). At small A, there is no empty core at the outlet of the nozzle, pressure in the core is no longer equal to p_2, and the problem becomes indeterminate.

Figure 5 compares all of the above-discussed results in the form of the dependence of μ and $\overline{R}_2$ on the inverse twist of the flow 1/A: ABR denotes the PMD of Abramovich; LUG denotes the calculation by Lugovtsov, PT denotes the potential twist, and RBM denotes rigid-body motion.

The highest values of μ are obtained for the PMD, the lowest for the LUG. For large twists (small 1/A), all of the calculations give roughly the same results within the experimental error. More indicative are the values of the radius $\overline{R}_2$ of the core of the outlet. The values of this quantity are the same for ABR and PT, while the calculations in [1] yield unrealistically larger values of $\overline{R}_2$; the RBM $\overline{R}_2$ nearly coincides with the value of this quantity from the ABR and PT in the case of large twists; the RBM core disappears at $A \leq 1/2$. In the general case, the published experimental values of $\overline{R}_2$ are either above or below the ABR curve for $\overline{R}_2$. (It should be noted that in Fig. 2 of [1], three was a tendency to use a single series of measurements; with allowance for the errors of these measurements, they agree in equal measure with the LUG $\overline{R}_2$ and the ABR $\overline{R}_2$.)

It should be noted that the use of integral theorems of mechanics alone is insufficient to solve the problem of a centrifugal nozzle in the completely nonlinear formulation. They must be augmented by some kind of heuristic conditions of nonambiguity. In all of the schemes examined above, we made the important assumption that the flow became cylindrical at the outlet ($r = R_2 = \text{const}$). Also, Lugovtsov [1] used the physically incorrect condition $B_2 = B_1$; all of the other relations in Fig. 5 correspond to the maximum discharge.

Thus, the "exact" solution in [1] should be considered incorrect, and the doubts cast on the reliability of the PMD are unjustified.

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PRINCIPLE OF MAXIMUM DISCHARGE

B. A. Lugovtsov

It was shown in [1] that discharge in a swirl sprayer with a Borda mouthpiece is based on conservation laws and does not agree with the value established by means of the PMD. This conclusion casts doubt on the overall validity of the PMD for determining flow parameters in centrifugal nozzles, spillways, and similar flow channels.

Stepanov (see [2]) asserts that the results presented in [1] were erroneous. However, this was actually not the case that was made. For example, it was stated the the flow scheme examined in [1] was erroneous. Without proving this, the author then proceeds to consider a different scheme which bears no relationship to the flow in a swirl sprayer.

Stepanov writes: "A suction force exists at sharp edges in a continuous potential flow, and this force should be incorporated into Eq. (15)." However, in [1] the object study was not a continuous potential flow, but a potential flow with a closed separation zone (region) which arose at the sharp inside edge of a cylindrical nozzle. The structure of the flow in this region is determined solely by the requirement that the velocity be finite and, thus, that there be no suction force. The flow structure in a closed separation region cannot be determined unambiguously within the model of an ideal incompressible fluid. This fact, however, does not preclude the effective use of conservation laws.

Stepanov goes on to write that "As is known from the hydraulic theory of spillways with a wide ramp...the PMD corresponds to the critical flow and follows from the Euler equation." This statement is accurate only in the sense that, under certain conditions and in certain cases, the PMD makes it possible to approximately determine flow parameters. However, it is possible to cite numerous examples where the PMD gives erroneous results for flows in spillways and similar channels.

Let us examine the flow in the spillway depicted in Fig. 1. Discharge across an infinitesimally thin horizontal ramp OC occurs from an infinitely deep reservoir with a quiescent liquid. Meanwhile at an infinitely distant point A on the free surface, the level of the quiescent liquid is no greater than the level H of the ramp OC. Within the framework of the model of an ideal incompressible fluid, it is natural to suggest that the flow is a potential flow except for a closed separation region which develops in the neighborhood of the sharp edge O. There is no suction force present (the boundary streamline of the separation zone has a horizontal tangent at point O). The structure of the flow in this region is unimportant to the subsequent discussion. Uniform flow is presumed to take place at infinity above the ramp. The discharge down such a spillway can be found by means of the laws of conservation of mass, momentum, and energy. As a result, we have

$$Q = uh = \frac{2}{3\sqrt{3}} H \sqrt{gH}, \quad u = \frac{2}{\sqrt{3}} \sqrt{gH}, \quad h = \frac{1}{3} H.$$

If we use the PMD, we find that

$$Q = \frac{2}{3} \sqrt{\frac{2}{3}} H \sqrt{gH}, \quad u = \sqrt{\frac{2}{3}} \sqrt{gH}, \quad h = \frac{2}{3} H.$$

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